

## GE 210 MIDTERM EXAM QUESTIONS 2004

**Time: 2 1/2 hours.**

**Students are allowed to bring in 1 sheet of formulae, etc., but no solved problems. Calculators are allowed. There are five questions, each having equal marks.**

1. A certain sessional lecturer wants to ensure that his students know how to solve problems that require them to use first principles. The students have studied the probabilities of possible outcomes for the roll of two dice, each die having six sides, and the toss of a coin having two sides. To make the study more interesting, the sessional has asked his students to consider a two-stage situation. In the first stage, the coin is tossed. If the outcome is a tail, the result is zero. If the outcome is a head, the second stage occurs and two dice are rolled. The result is the sum of the two upturned faces of the dice. For the set of possible results:
  - a) Draw the probability distribution.
  - b) Determine the lower quartile, median, the upper quartile and the mode.
  - c) Calculate the mean, variance and standard deviation.
  - d) Calculate the coefficient of variation.
2. A library employee shelves a large number of books every day. The average number of books mis-shelved per day is estimated over a long period to be 2.5. Calculate:
  - a) The probability that exactly 3 books are mis-shelved in a particular day.
  - b) The probability that between 5 and 10 books (inclusive) are mis-shelved in a 4 day period.
  - c) The probability that fewer than 2 books on one day and more than 2 books on the next day are mis-shelved.
3. Each automobile leaving a certain motor company's plant is equipped with five tires of a particular brand. Tires are assigned to cars randomly and independently. The tires on each of 100 such automobiles were examined for major defects with the following results:

No. of tires with defects	No. of automobiles (occurrences)
0	75
1	18
2	4
3	2
4	0
5	1

- a) Calculate the probability that a randomly selected tire from this manufacturer will contain a major defect.

- b) Suppose you buy an automobile of this make. From the results of a) calculate the probability that it will have at least one tire with a major defect.
  - c) What is the probability that, in a fleet purchase of six of the cars, at least half the cars have no defective tires?
  - d) What is the expected number of defective tires in the fleet purchase of six cars?
4. As  $t$  increases from 0, the probability density for a certain event constantly decreases to zero over the range  $0 \leq t \leq 2$ , and is 0 otherwise
- a) Determine the probability density function for the event. Show that this probability density function meets all requirements for a probability distribution.
  - b) For the probability density function  $f(x) = x/2$  over the range  $0 \leq x \leq 2$ , and 0 otherwise:
    - i) Determine the mean and standard deviation.
    - ii) Write the equation for the cumulative probability distribution.
    - iii) Calculate  $P(0.8 \leq x \leq 1.4)$ .
5. A survey of customers of an auto repair shop with more than one mechanic showed that 20% had complaints about the service. Even though Mechanic A performs only 40% of the jobs in the shop, 50% of the complaints were about him.
- a) If a job is done by Mechanic A, what is the probability that there will be a complaint?
  - b) If, for a particular job, there is no complaint, what is the probability that the job was not done by Mechanic A?
  - c) The workload in the shop is high, so the manager of the shop hires another mechanic. This reduces the workload by 25% for each of the original mechanics. The manager finds that she now has complaints about 30% of the jobs. If the new mechanic were to carry out a repair to your vehicle, what is the probability that it would be done satisfactorily? Assume that the performance of the original mechanics does not change, i.e. the probability of complaint about each mechanic does not change.

**END OF EXAM**